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Understanding the complexity of the fracture mechanics of brittle matrix composites was a subject of several analytical studies in this work. The studies included finding the effect of imperfect fiber-matrix interfaces, both frictional and nonhomogeneous, on the fracture mechanics of composites; readdressing and questioning the current criteria for crack deflection at fiber-matrix interfaces used for developing optimum composite interfaces; comparing axisymmetric and planar approximations of three dimensional fracture mechanics problems, since stiffness reduction and fracture toughness of composites are evaluated using such assumptions. The Material Directorate at Wright Patterson Air Force Base used these studies for verifying their computational and experimental models.

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**FRACTURE MECHANICS OF BRITTLE MATRIX COMPOSITES  
WITH IMPERFECT INTERFACES**

**by**

**Autar K. Kaw  
Principal Investigator  
Mechanical Engineering Department  
University of South Florida, Tampa, FL 33620-5350**

**Final Technical Report Submitted**

**to**

**Air Force Office of Scientific Research  
AFOSR/NA  
Washington, DC 20332  
Dr. Walter F. Jones, Monitor  
Award Number: F49620-92-J-0542**

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## **ABSTRACT**

Understanding the complexity of the fracture mechanics of brittle matrix composites was a subject of several analytical studies in this work. The studies included finding the effect of imperfect fiber-matrix interfaces, both frictional and nonhomogeneous, on the fracture mechanics of composites; readdressing and questioning the current criteria for crack deflection at fiber-matrix interfaces used for developing optimum composite interfaces; comparing axisymmetric and planar approximations of three dimensional fracture mechanics problems, since stiffness reduction and fracture toughness of composites are evaluated using such assumptions. The Material Directorate at Wright Patterson Air Force Base used these studies for verifying their computational and experimental models.

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## INTRODUCTION

The advantages of using ceramics are their high refractoriness and strength, low density, excellent corrosion and oxidation resistance, and low cost. However, their brittle nature and low fracture toughness makes them prone to catastrophic failure. A promising method to overcome these shortcomings is by reinforcing ceramics by aligned continuous fibers, such as, silicon carbide and carbon. The resulting composite materials are called ceramic matrix composites (CMCs), and have increased fracture toughness and fail more gracefully. However, they are highly anisotropic and fail in a complex manner, and often do not follow the basic concepts of fracture mechanics. So, the basic understanding and modeling of the mechanics of these materials are essential for the development of these materials.

In this work, we studied several issues concerning the mechanics of fracture of these materials. These included finding the effect of imperfect fiber-matrix interfaces, both frictional and nonhomogeneous, on the fracture mechanics of composites. We also studied several general issues. These included readdressing the current criterion for crack deflection at fiber-matrix interfaces for guiding material scientists to develop optimum CMCs. We also compared axisymmetric and planar approximations of analytical fracture mechanics models, since both are widely used in literature for recommending design guidelines.

All of the work done in this grant is available in refereed journals. The titles, abstracts and their references are given in Appendix A. The cumulative list of researchers is given in Appendix B. We give a comprehensive executive summary of the significant work accomplished in this grant in the next chapter.

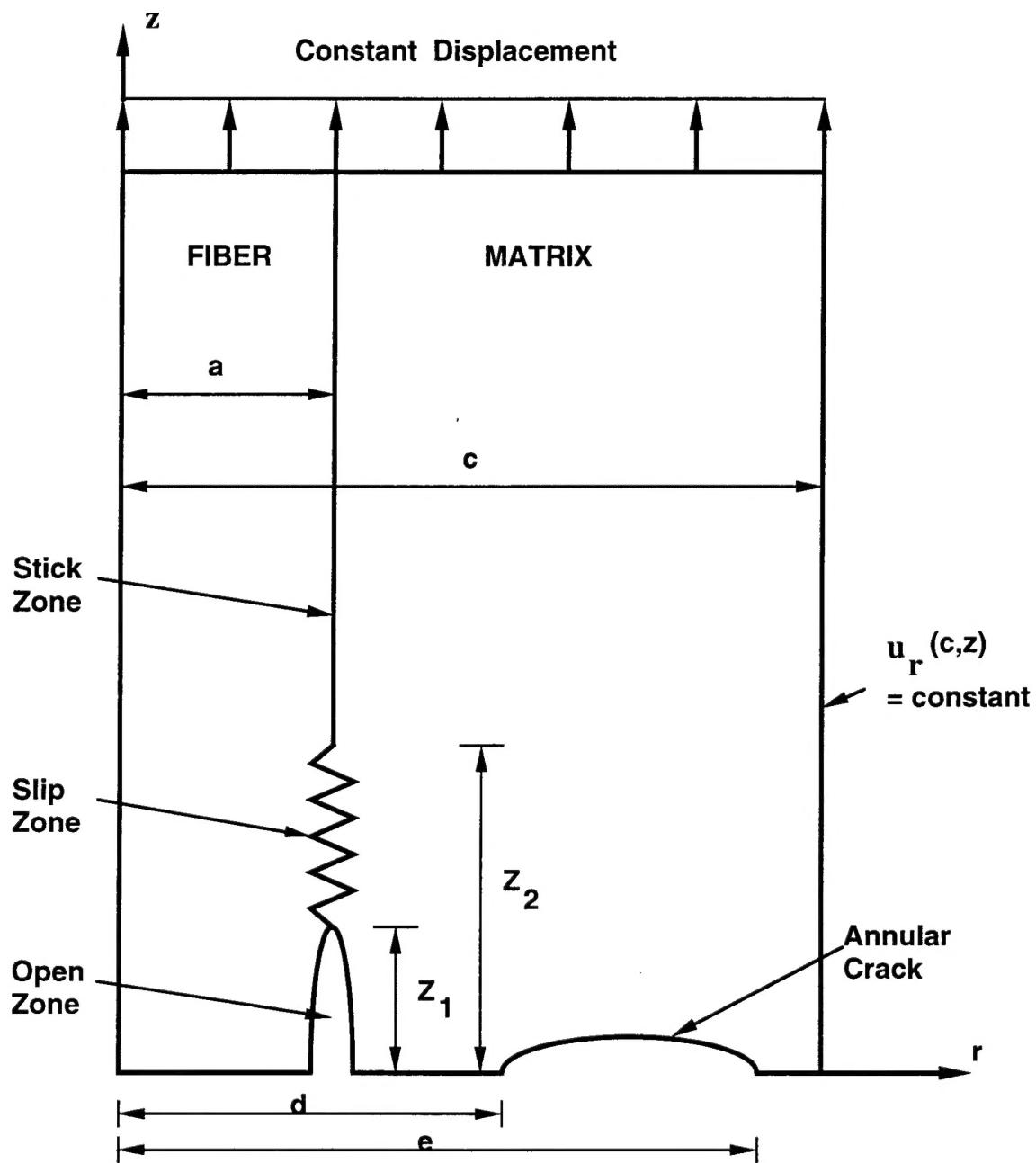
## TASKS

**Task 1: Stress fields in a ceramic matrix composite in presence of frictional fiber-matrix interfaces (Kaw, Kunchithpatham, and Pagano, 1995)**

Consider a unidirectional ceramic matrix composite subjected to an axial strain along the fiber direction. The cracks will first develop in the matrix due to its lower failure strain than that of the fiber. When a matrix crack reaches the interface of the fiber and the matrix, the interface may open or slip. This opening/slipping of the interface blunts the crack, and slows and arrests the propagation of the crack. Although this blunting of the crack increases the fracture toughness of the composite, the damage in the interface reduces the axial compressive and transverse strength of the composite. These conflicting effects of interfacial damage make it important fully to understand the mechanics of matrix fracture in ceramic matrix composites as a function of material, and geometrical and loading parameters.

The model used in this study is a concentric cylinder geometry with a frictional fiber-matrix interface (Figure 1). An annular matrix crack is assumed in the matrix, while the interface follows the Coulomb friction law. We assume a constant temperature change due to processing. A uniform remote axial strain represents the external applied load. Using all the equations of elasticity for each constituent of the composite cylinder and the Coulomb friction law at the interface, we obtain the solution as coupled integral equations and inequality conditions.

The model developed here is a continuing effort to relax the assumptions made in models available in the literature. The assumptions relaxed in our model allowed us to study the fracture mechanics of CMCs with nondilute fiber volume fractions, an annular matrix crack of any length and location, mismatches of the linear coefficients of thermal expansion of the fiber and the matrix, and satisfaction of all equations of elasticity for the fiber and the matrix.



**Figure 1.** Schematic of a representative volume element of a brittle matrix composite with a frictional interface and annular matrix crack under a thermomechanical load.

In the numerical results for this study, the annular crack in the matrix is considered a through crack. Also, we assumed that the coefficient of thermal expansion of the matrix was greater than that of the fiber and that the Poisson's ratios were equal. A SiC/CAS material system with the following thermoelastic properties, geometrical parameters and temperature change was considered for the numerical results.

**Table 1. Properties of SiC/CAS System**

Property	Units	SiC Fiber	CAS Matrix
Young's Modulus	GPa	207	98
Poisson's Ratio		0.25	0.25
Coefficient of Thermal Expansion	m/m/ <sup>0</sup> C	3.5x10 <sup>-6</sup>	6.5x10 <sup>-6</sup>

Fiber Volume Fraction = 0.40%

Temperature Difference = -1000<sup>0</sup>C

We observed the following for the above SiC/CAS material system under a remote axial strain load.

- The length of the slip zone is dependent only on the dimensionless ratio between the remote radial pressure at the interface and the remote matrix axial stress. Different combinations of temperature difference, remote axial strain and coefficients of thermal expansion may result in the same value of this dimensionless ratio.  
The length of the slip zone is a monotonically increasing function of the remote mechanical load and a monotonically decreasing function of the coefficient of friction.
- The predicted lengths of the slip zone are similar to those of approximate shear-lag

analysis type models (Gu and Manganon, 1992), only for large slip lengths (Figure 2). This validates the use of the shear-lag model for large slip lengths.

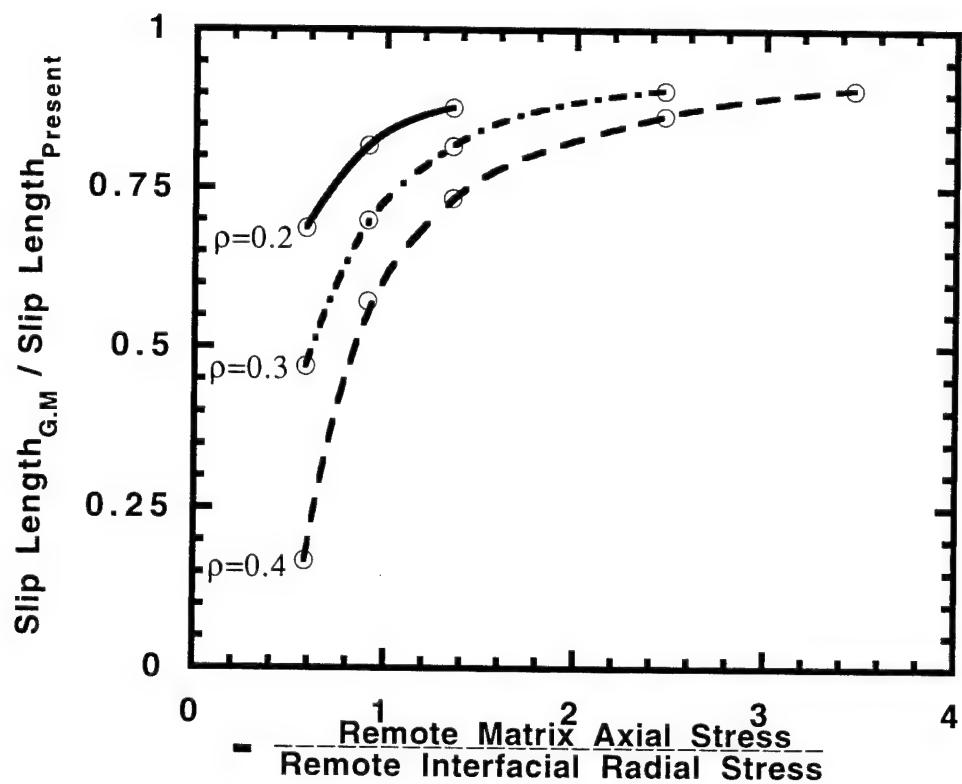
- The axial stresses in the fiber and matrix are independent of the radial coordinate away from the transverse crack plane and the slip-stick transition zone (Figure 3). The shear and the radial stresses in the slip zones are fairly constant for low coefficients of friction and may validate constant shear stress assumption models in these cases. These stresses also are nearly independent of the remote mechanical load.

The maximum tensile axial and hoop stresses in the matrix are equal to the respective axial and hoop stresses in the uncracked composite (Figures 4 and 5). Assuming some tensile failure stress criteria, the matrix crack and the interface damage may not influence development of new matrix cracks.

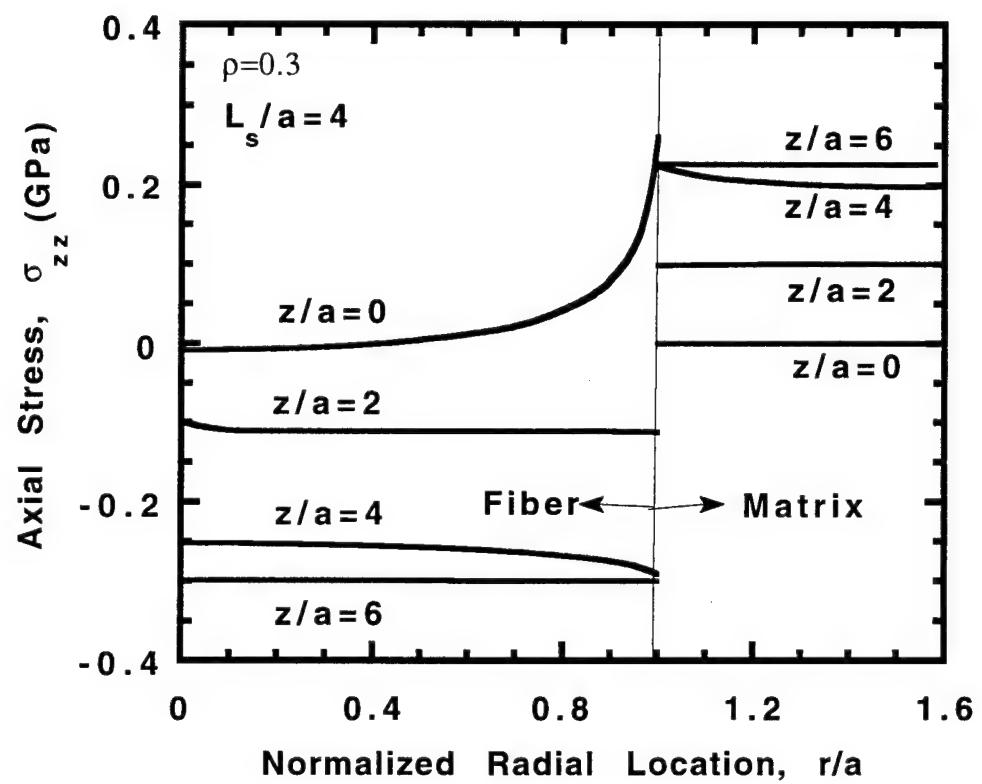
**Task 2: Fracture Mechanics of Composites with Nonhomogeneous Interphases and Nondilute Fiber Volume Fractions (Bechel and Kaw, 1994)**

Many micromechanical planar models of composites have been developed to understand the fracture mechanics of composites. These planar models have considered a fiber crack in composites with a dilute fiber volume fraction (Gupta, 1973), periodic cracks in composites with a nondilute fiber volume fraction (Erdogan and Bakioglu, 1976), and composites with nonhomogeneous interphases (Delale and Erdogan, 1988). The planar models do not explain the effect of nondilute fiber volume fractions on the fracture mechanics of a composite with an isolated fiber or matrix crack, the effect of various nonhomogeneties in the interphases and irregular fiber spacing on the fracture mechanics of composites.

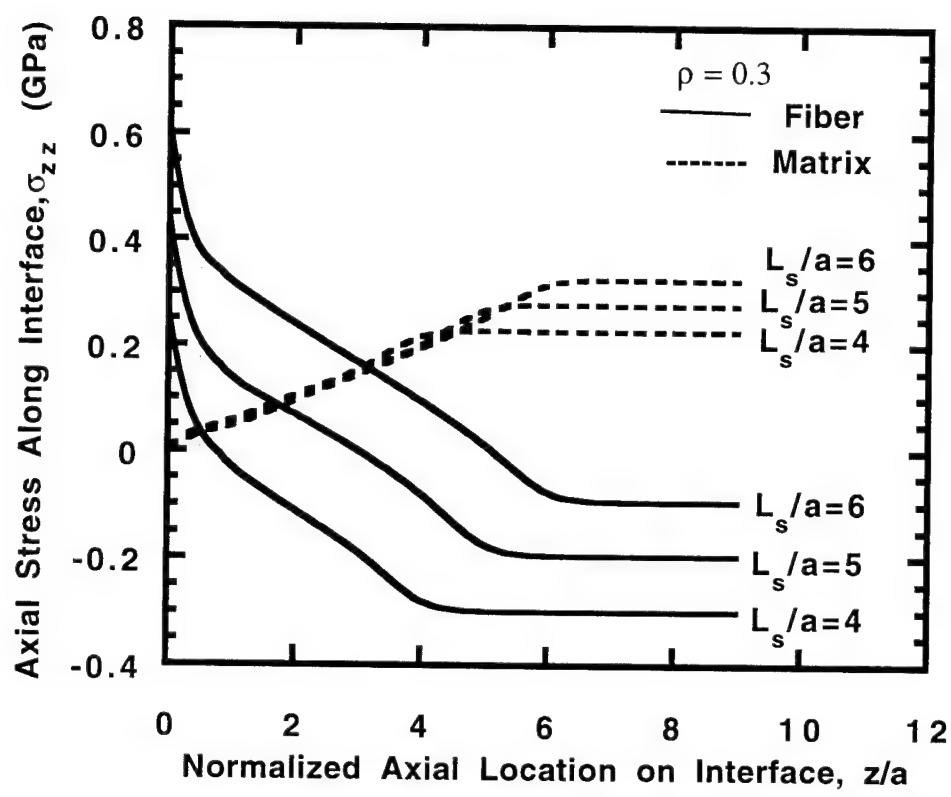
We have addressed the preceding questions in this work. The geometry of the problem is shown in Figure 6. The composite consists of  $(2n-1)$  planar strips of infinite



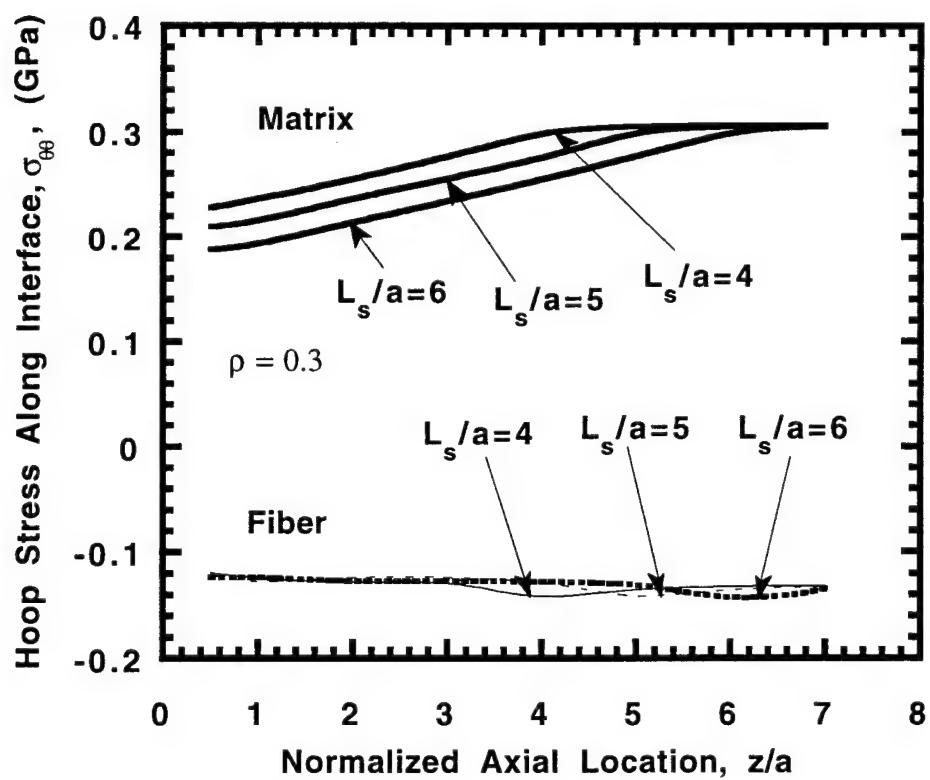
**Figure 2.** Ratio of slip length of Gu and Mangonon type model to present model as a function of the negative of the ratio of remote axial matrix stress to remote interfacial radial stress for constant coefficients of friction.



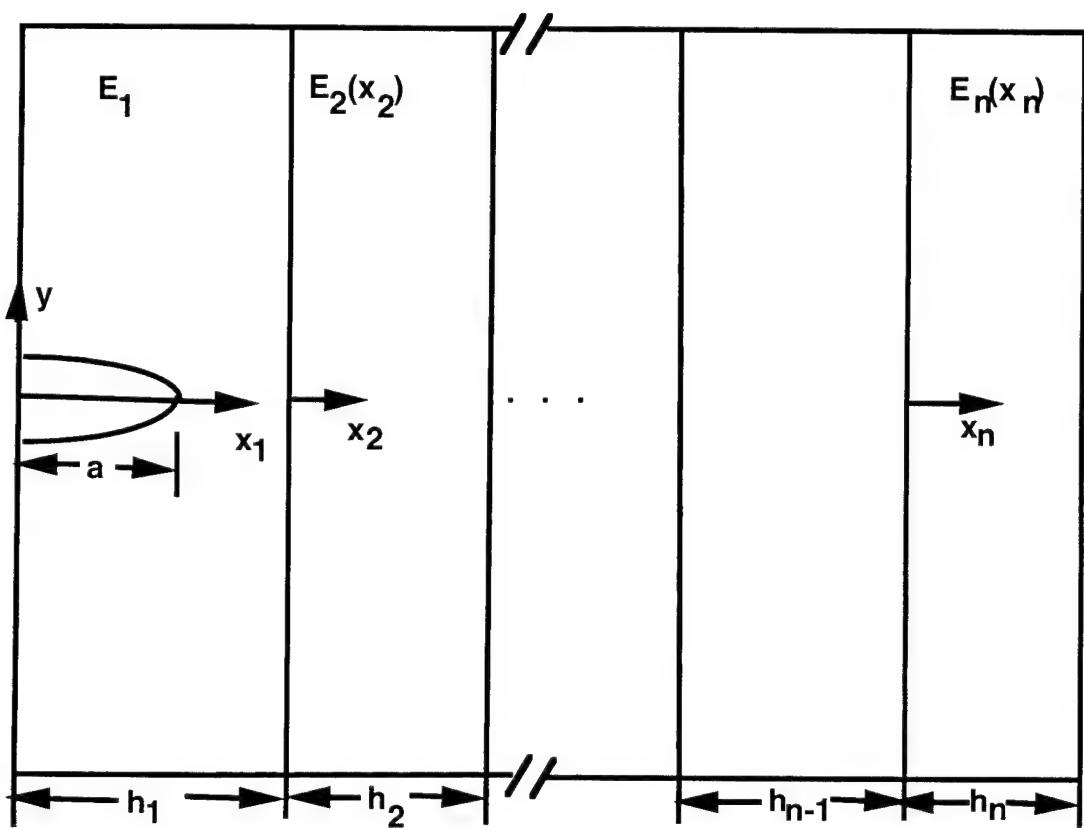
**Figure 3.** Fiber and matrix axial stresses as a function of the normalized radial location for constant normalized axial locations.



**Figure 4.** Interfacial axial stresses as a function of normalized axial location for constant slip lengths.



**Figure 5.** Interfacial hoop stresses as a function of normalized axial location for constant slip lengths.

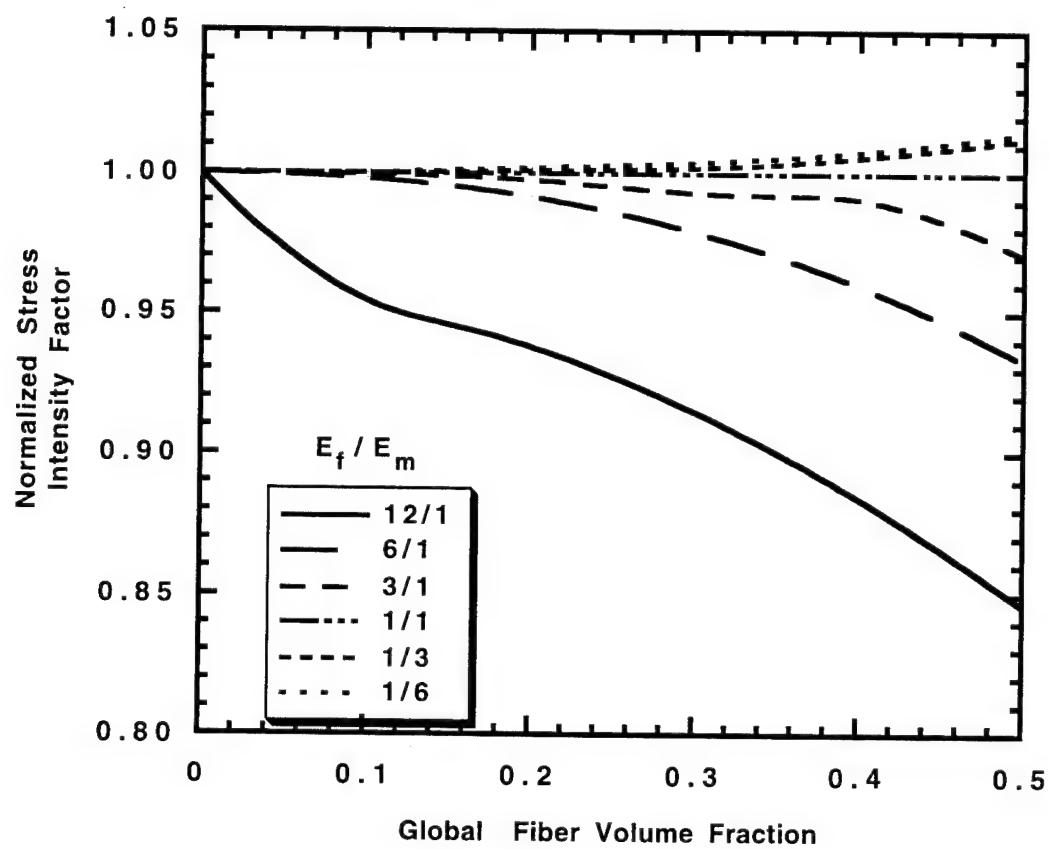


**Figure 6.** Geometry of a composite with a single damaged layer.

length and finite width that are perfectly bonded, isotropic and linearly elastic. A strip in the center of the composite has a symmetric crack and approximates a damaged fiber or matrix region. The crack may touch the interface of the cracked layer and the first undamaged layer. The remaining undamaged fibers, the interphases, and the matrix are approximated by  $(2n-1)$  infinite strips. The Young's modulus and Poisson's ratio of each undamaged strip may follow any variation through its thickness. The layered composite model is loaded with a remote uniform axial tensile strain.

The main conclusions of this study were:

- A crack is less likely to grow into a nonhomogeneous interphase that has a greater average elastic moduli near the interface of the cracked layer and the interphase.
- The fiber and matrix layers away from the damaged layer affect the critical stresses and the stress intensity factor at the crack tip. However, the ratios of the interfacial to the axial stresses at the crack tip, which determine crack deflection, are dependent only on the relative elastic moduli of the damaged and adjacent media. These ratios are independent of the geometry and the elastic moduli of the materials away from the damaged layer.
- The stress intensity factor at the crack tip of a damaged fiber layer decreases with an increase in the fiber volume fraction and an increase in the fiber to matrix moduli ratio (Figure 7). If the damaged fiber layer is less stiff than the matrix, the fiber volume fraction affects the stress intensity factor.
- The processing techniques for composites produce internal defects, such as micro cracks and uneven fiber spacing. Some fibers get placed further from each other than the average fiber distance, while other fibers may be placed closer together. Matrix crack initiation stress is found approximately to halve when the local fiber



**Figure 7.** Normalized stress intensity factor as a function of global fiber volume fraction for constant fiber to matrix elastic moduli ratio.

spacing doubles. Barsoum, et al (1991) made the same observation in their experimental study. On the other hand, for a transversely loaded composite, the most likely crack path is between closely packed fibers (Xu, et al, 1992). So, the uniform spacing of fibers is critical in increasing the matrix cracking initiation stress.

**Task 3: Comparison of Axisymmetric and Planar Fracture Mechanics Models for Fiber Reinforced Composites (Kaw and Ye, 1994; Kaw and Jadhav, 1994)**

In the past two decades, several analytical models have been developed not only to understand the fracture mechanics of fiber reinforced composite materials, but also to evaluate mechanical properties of a composite. However, since complete analytical solution of three-dimensional fracture mechanics problem in a composite geometry is generally not tractable, they are based on either axisymmetric or planar assumptions. In this work, we studied how these assumptions affect the evaluation of the mechanical properties of the composite.

The mechanical properties found from analytical models with planar or axisymmetric assumptions include matrix crack initiation stress (Wang, et al, 1992), longitudinal stiffness reduction due to multiple matrix cracks (Kaw and Gadi, 1993), crack deflection criteria at the fiber-matrix interface (Cornie, et al, 1991; He and Hutchinson, 1989), and the size of damage in fiber-matrix interfaces (Schwietert and Steif, 1991). Three fundamental parameters - stress intensity factors at the crack tips, crack opening displacements, and stresses at the fiber-matrix interface are used to evaluate the above critical mechanical parameters. In this work, we compare these three fundamental parameters for the two types of models to raise and answer questions about the validity of using the planar or axisymmetric model to quantify the above mechanical properties.

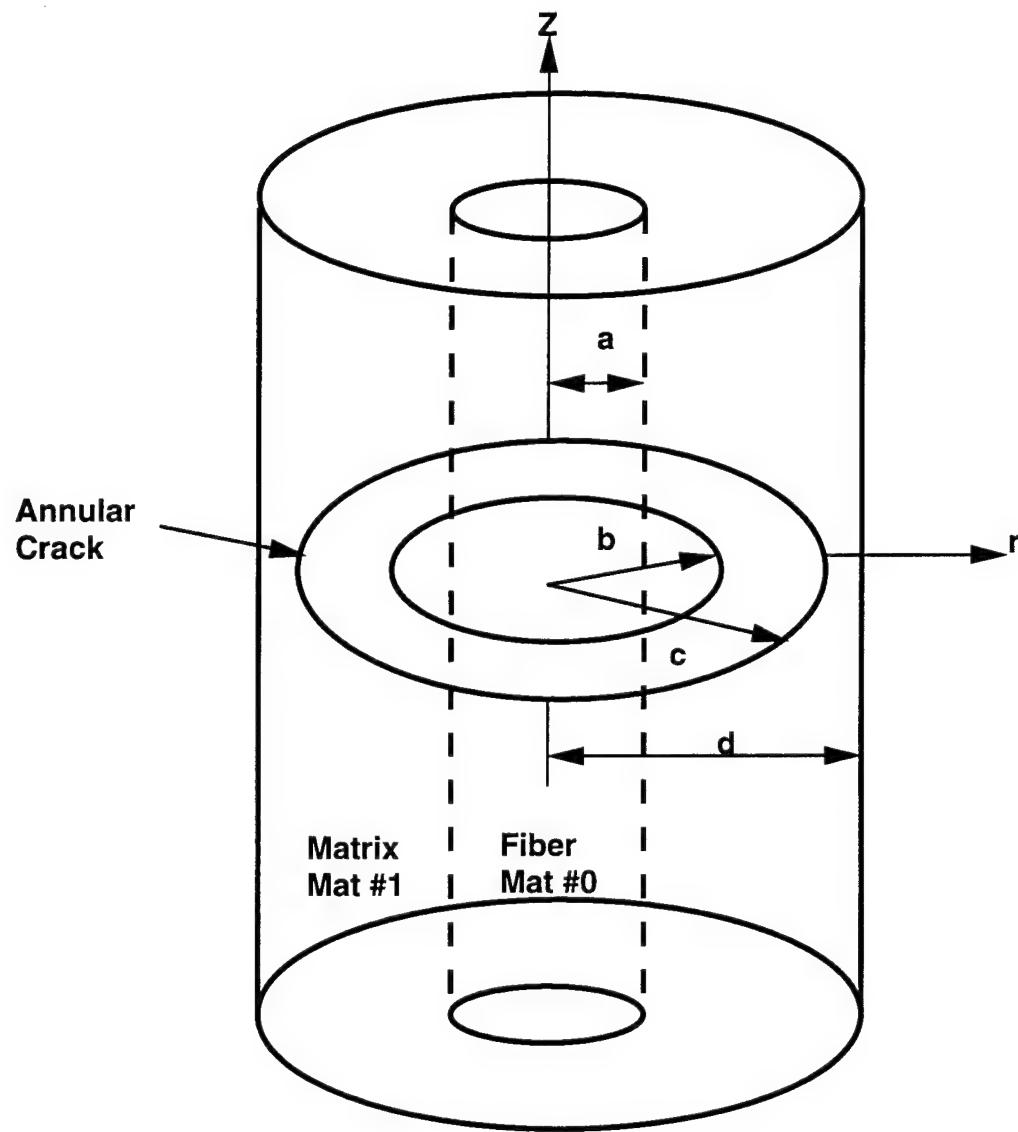
Axisymmetric and planar geometries approximate a composite in this work. The axisymmetric model consists of an infinitely long solid cylinder (fiber) perfectly bonded to a long hollow cylinder (matrix) as shown in Figure 8. The matrix consists of an annular crack of length ' $c-b$ ', ( $a \leq b < c \leq d$ ) at  $z=0$ . In the planar model, an infinitely long finite width strip is bonded to two symmetric infinitely long and finite width strips (Figure 9). The matrix has a crack of length ' $C-B$ ', ( $A \leq B < C \leq D$ ). In both models, the fiber and the matrix are linearly elastic, isotropic and homogeneous, but with distinct elastic properties. The crack in the matrix is subjected to a constant pressure, and the bond between the fiber and the matrix is considered perfect.

The main results of this study were

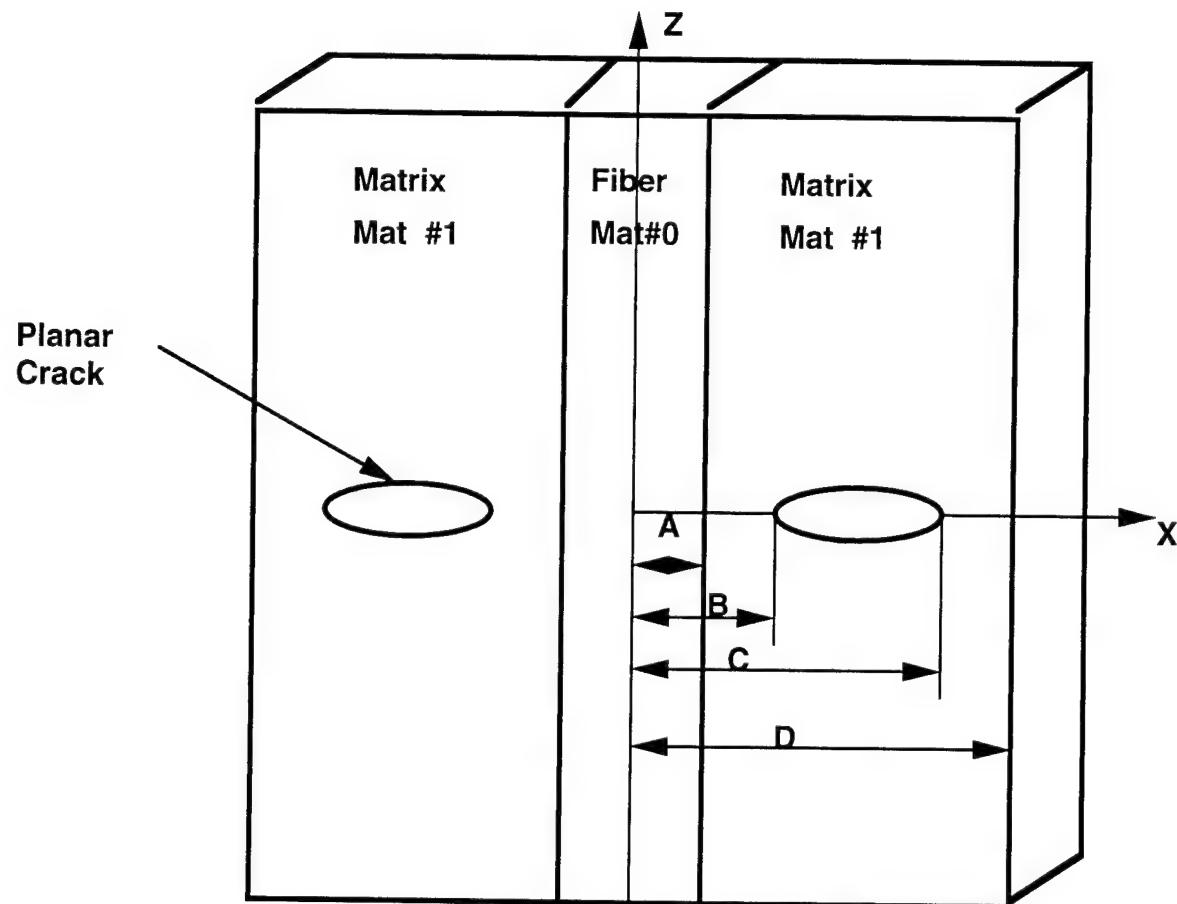
- The predictions about whether a crack deflects along the interface or penetrates the adjoining material are similar in the axisymmetric and planar cases. The crack deflection criteria by Cornie, et al (1991) is used to show this as follows.

The peel (shear) stress ratio for the axisymmetric case is defined as the ratio of the maximum radial (shear) stress at the interface to the maximum axial stress in the fiber. According to a crack deflection criterion of Cornie, et al.(1991), if these ratios are greater than the ratio of interfacial radial (shear) to fiber axial strength, then crack deflection is assumed to take place at the interface. The argument is similar in the case for the planar problem. These ratios for the axisymmetric and planar peel (shear) stress are close to each other as the crack approaches the interface. This infers that the predictions about the nature of crack deflection would be similar in axisymmetric and planar models (Figures 10 and 11).

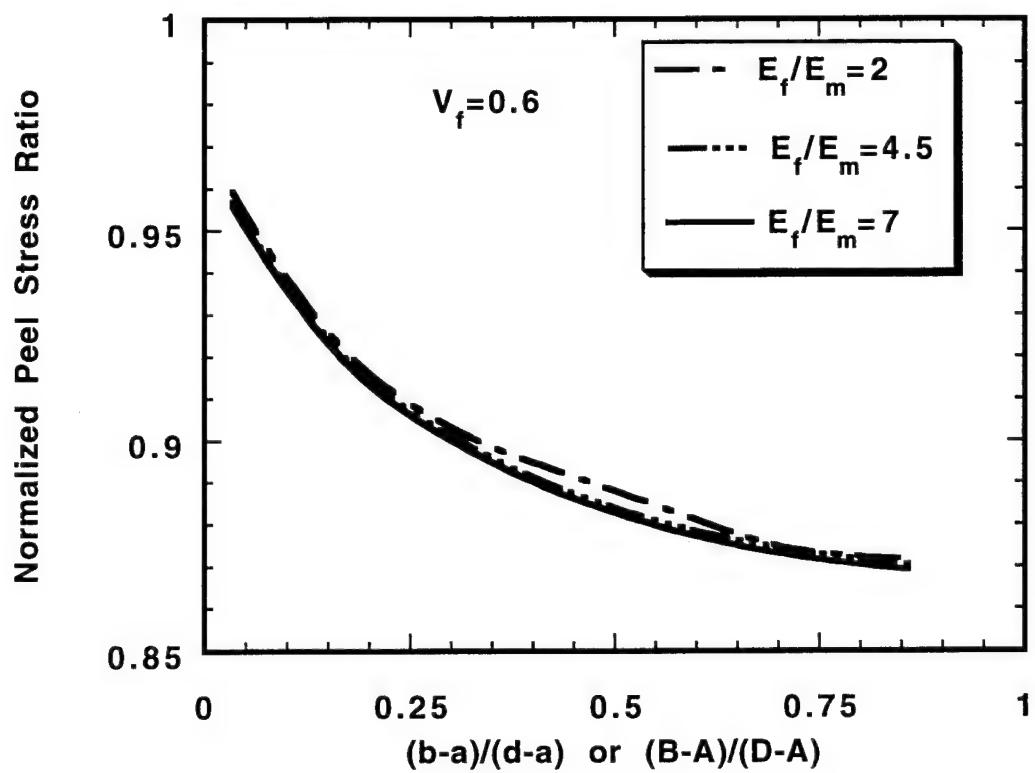
- Stress intensity factors and crack opening displacements differ considerably in the axisymmetric and the planar cases. For example, Figure 12 shows the ratio of the annular to planar stress intensity factors, as a function of normalized crack length



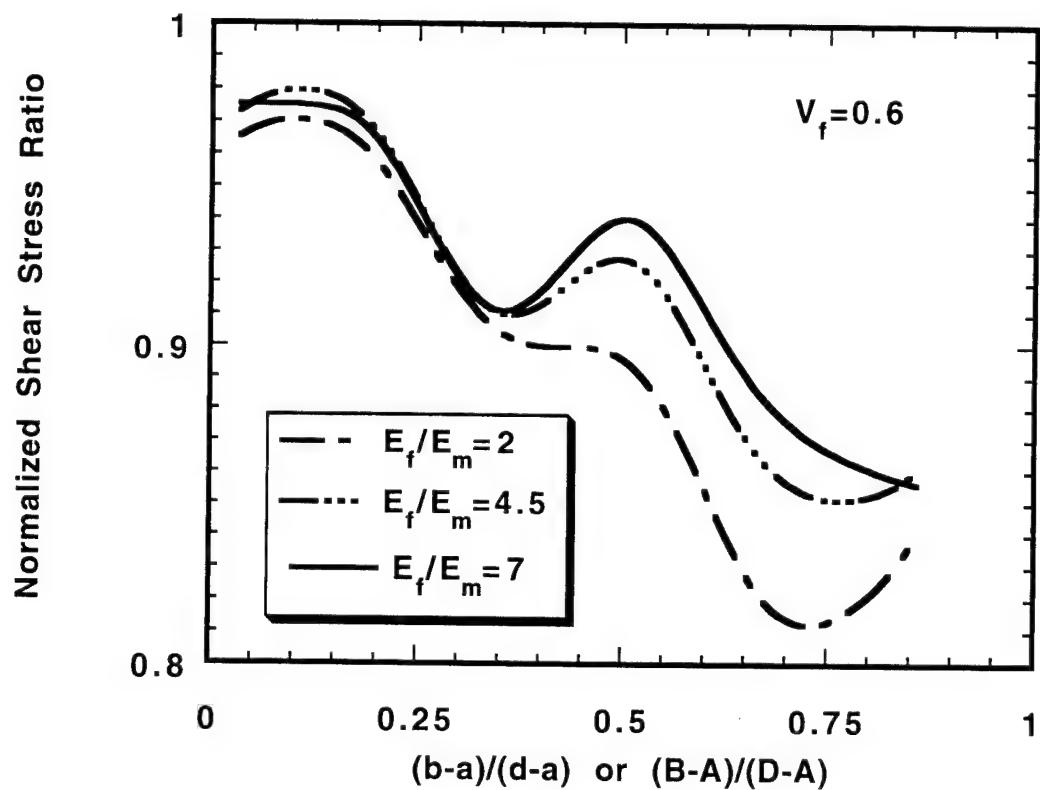
**Figure 8.** Schematic of an annular matrix crack in an axisymmetric composite geometry.



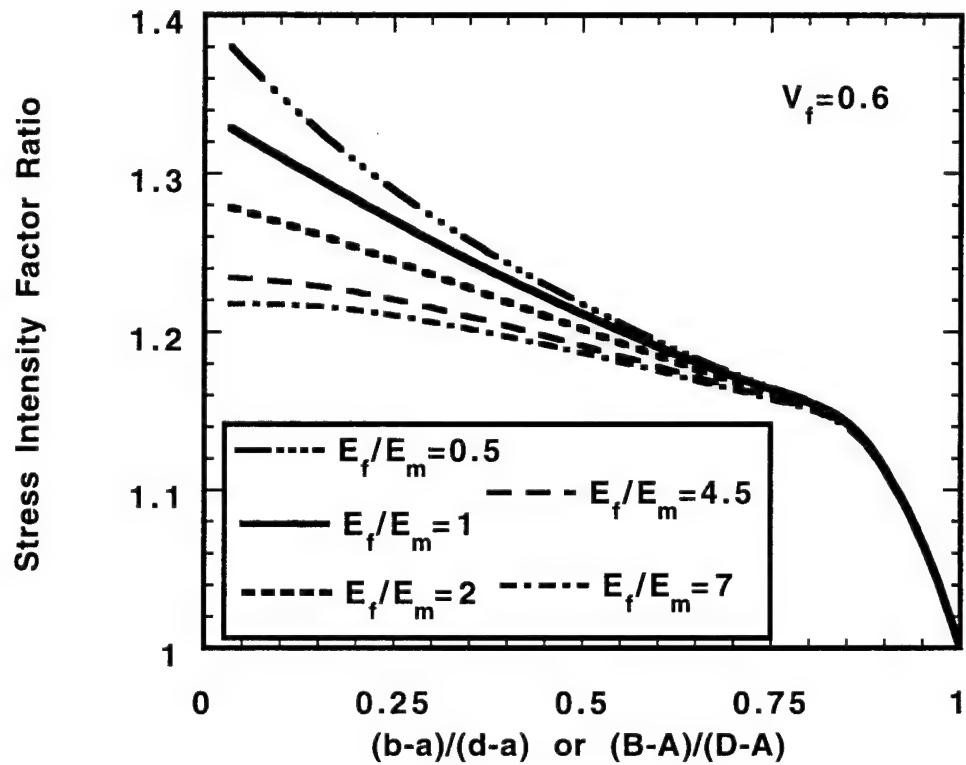
**Figure 9.** Schematic of a planar matrix crack in a planar composite geometry.



**Figure 10.** Ratio of annular to planar (plane strain) normalized peel stress as a function of normalized crack location for constant fiber to matrix moduli ratio (zero outer edge normal displacement).



**Figure 11.** Ratio of annular to planar (plane strain) normalized shear stress as a function of normalized crack location for constant fiber to matrix moduli ratio (zero outer edge normal displacement).



**Figure 12.** Ratio of annular to planar (plane strain) stress intensity factor as a function of normalized crack location for constant fiber to matrix moduli ratio (zero outer edge normal displacement).

$[(b-a)/(d-a)$  (axisymmetric) or  $(B-A)/(D-A)$  (planar)]. For small cracks of the same length, the stress intensity factors are equal. However, as the crack approaches the interface, the stress intensity factor predicted by the annular model continues to increase more than that of the planar case as the fiber to matrix moduli ratio decreases. Since matrix crack initiation stress, stiffness reduction and strain energy release rates are a function of stress intensity factors and crack opening displacements, the prediction of these parameters will be considerably different in the planar and the axisymmetric case. This difference increases with a decrease in the fiber to matrix moduli ratio and an increase in the fiber volume fraction.

**Task 4: A Criterion to Find Subsequent Direction of Crack Impinging at a Fiber-Matrix Interface (Pagano and Kaw, 1994)**

A fundamental problem important in the analysis of ceramic matrix composites (CMC), such as ceramic or glass-ceramic matrix composites, is the response of such materials in the presence of a crack impinging on an interface. Sometimes, additional layers or annular cylindrical regions of coating substances are introduced to provide enhanced damage tolerance, to redirect the crack such that it propagates in a benign manner, or to modify the physical and/or chemical behavior.

In this work new information is presented, which may serve as a tentative criterion for the direction of subsequent crack propagation. This criterion may provide, for example, a possible screening device for constituent materials selection. The concept used here is analogous to that derived by Cornie, et al, (1991) who assumed that whether a crack grows along the interface or penetrates into the next material is dependent on the respective ratio

of the strengths as following.

- The peel stress ratio is defined as the ratio of the radial stress at the interface to the axial stress in the fiber. If this ratio is greater than the ratio of interfacial radial to fiber axial strength, then debonding is assumed to take place at the interface. Mathematically, looking at the crack impinging on a bimaterial interface in Figure 13, debonding of the interface is assumed if

$$R_p = \frac{\sigma_{xx}^2(r=0, \theta=90^\circ)}{\sigma_{yy}^2(r=0, \theta=0^\circ)} > \frac{\sigma_p^i}{\sigma_{ult}^2} \quad (1)$$

where

$\sigma_p^i$  = peel strength of the interface, and

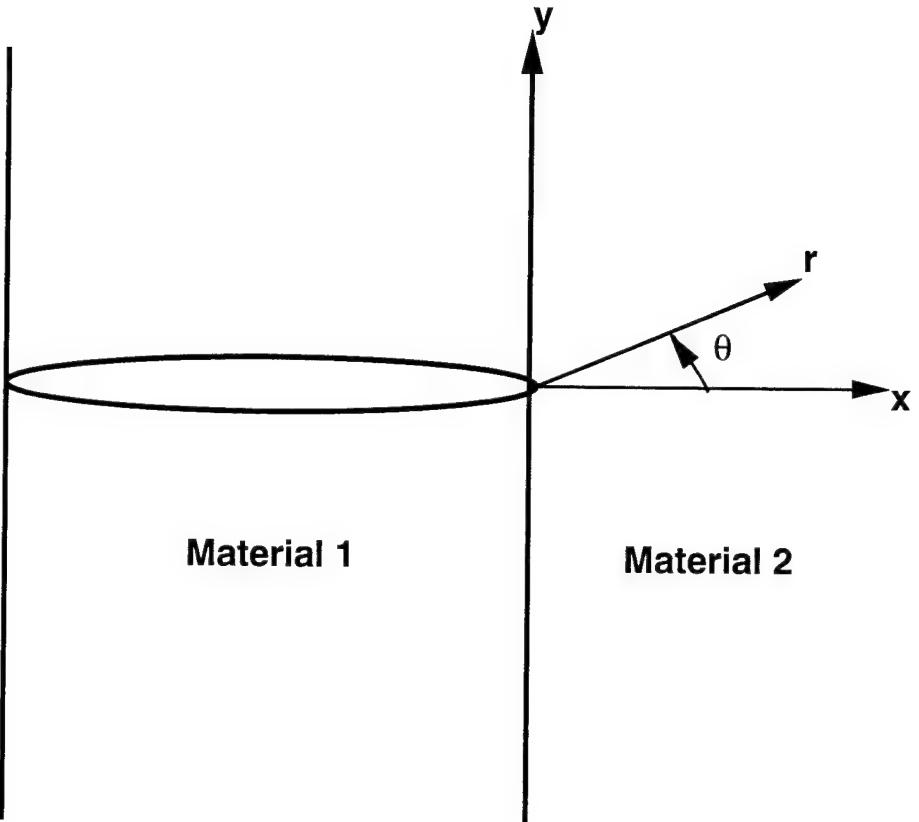
$\sigma_{ult}^2$  = ultimate strength of Material 2

- The shear stress ratio is defined as the ratio of the shear stress at the interface to the axial stress in the fiber. If this ratio is greater than the ratio of the interfacial shear strength to the fiber axial strength, then pull out of fibers is assumed to take place at the interface. Mathematically, looking at the crack impinging on a bimaterial interface in Figure 13, pull out of fiber is assumed if

$$R_s = \frac{\sigma_{xy}^2(r=0, \theta=90^\circ)}{\sigma_{yy}^2(r=0, \theta=0^\circ)} > \frac{\sigma_s^i}{\sigma_{ult}^2} \quad (2)$$

where

$\sigma_s^i$  = shear strength of the interface.



**Figure 13.** Schematic of a transverse crack impinging on a bi-material interface.

At the crack-tip,  $r=0$ , infinite stresses are predicted. In fact, the state of stress at  $r=0$  is ambiguous in that it depends on angle  $\theta$ . This has caused some to categorize the stress state as that of an unsymmetric stress tensor (Reissner, 1944). Others have used the polar coordinate representation of the singular stresses, which tends to mask the non-uniqueness of the state of stress since these components generally vary even at a non-singular point. In any event, the theory of elasticity breaks down, at least locally. This situation is not limited to crack tips in composite materials but also occurs in homogeneous materials. Despite this fact, the important branch of fracture mechanics has been developed and found to have practical utility in engineering and science. Thus the local ambiguity of the state of stress should not dissuade one from using the formulation to characterize physical behavior, such as failure, where the initial crack may deflect/reflect in an arbitrary direction. We ask the question: "What stress(es), if any, will serve as a criterion for this behavior?"

As an example of the new criterion suggested here, consider a composite such as Boron-Aluminum or Silicon Carbide-Glass with a ratio of the constituent shear moduli equal to 6, while the constituent Poisson ratios are both set equal to 0.25. Here the two stress ratios as given by equations (1) and (2) are

$$R_p = 0.4854$$

$$R_s = 0.1924$$

If one uses the polar coordinate representation of the form for the stresses instead of the Cartesian, one would replace the normal stresses,  $\sigma_{yy}^2(r=0, \theta=90^\circ)$  in equations (1) and (2) by the maximum hoop stress,  $\sigma_{\theta\theta}^2(r=0, \theta)|_{\max}$ . However, the maximum hoop stress occurs at  $\theta=0$  and the ratios given by equation (1) and (2) remain the same. However, the maximum hoop stress,  $\sigma_{\theta\theta}^2(r=0, \theta)|_{\max}$  or the normal stress  $\sigma_{yy}^2(r=0, \theta=0^\circ)$  are not the

principal normal stress. What happens when the maximum principal normal stress in the plane,  $\sigma_n^2(r=0, \theta)|_{\max}$  is used in the crack deflection criteria as

$$R_p = \frac{\sigma_{xx}^2(r=0, \theta=90^\circ)}{\sigma_n^2(r=0, \theta)|_{\max}} \quad (3)$$

$$R_s = \frac{\sigma_{xy}^2(r=0, \theta=90^\circ)}{\sigma_n^2(r=0, \theta)|_{\max}} \quad (4)$$

For the above material systems, these ratios then are

$$R_p = 0.2873$$

$$R_s = 0.3854$$

which is a decrease of a factor of 1.218. Although either criterion may or may not be valid, we have portrayed the non-uniqueness issue. Consistency with experimental data is the only way to verify which criterion is valid.

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**Appendix A. ABSTRACT OF PUBLISHED JOURNAL PAPERS**

**Title:** **Fracture Mechanics of Composites with Nonhomogeneous Interphases and Nondilute Fiber Volume Fractions**

**Authors:** **V.T. Bechel and A.K. Kaw**

**Journal:** **International Journal of Solids and Structures, Vol. 31, No. 15, pp. 2053-2070, (1994).**

**Abstract :** A linear elastic fracture mechanics model is developed for a periodic composite geometry with a single cracked layer under uniform longitudinal strain. The interface between the fiber and the matrix can be nonhomogeneous, homogeneous or perfect. Four fracture mechanics models are considered - 1) dilute fiber volume fraction composite with nonhomogeneous interphases, 2) periodic composite with nondilute fiber volume fraction, 3) periodic composite with irregular fiber spacing near a matrix layer, and 4) hybrid composites with more than one type of fiber. The stress intensity factors at the crack tips and the interface stress fields are studied to understand the fracture mechanics of composites as a function of the relative elastic moduli of the fiber, the matrix and the interphase, and the global and local fiber volume fractions.

**Title:** Comparison of Fracture Mechanics Models with Axisymmetric and Planar Assumptions

**Authors:** A.K. Kaw and J. Ye

**Journal:** Composites Engineering, Vol. 4, No. 6, pp. 621-636, (1994).

**Abstract:** Many fracture mechanics models based on either 3D-axisymmetric or 2D-planar models are being used to predict mechanical parameters of composites, such as, their fracture toughness, matrix crack initiation stress and stiffness. In this study, fracture mechanics models for a composite geometry with a matrix crack under uniform crack pressure are developed using both 3D-axisymmetric and 2D-planar assumptions. Three critical parameters, namely the stress intensity factor at the crack tips, the maximum crack opening displacements and the stress ratios at the interface, which are used to quantify the above mechanical parameters of a composite, are compared for the two models. The results for the stress ratios at the interface, which are used to predict the path of crack propagation near a fiber-matrix interface, are found to be close to each other. However, the results for the stress intensity factor and the crack opening displacements, which are used to quantify matrix crack initiation stress and damaged composite longitudinal stiffness, are found to differ considerably.

**Title:** Axisymmetric Elastic Response of a Composite Cylinder with a Broken Fiber

**Authors:** A. K. Kaw and D. Jadhav

**Journal:** Theoretical and Applied Fracture Mechanics, Vol. 21, pp. 197-206, (1994).

**Abstract:** An analytical solution to the problem of a penny-shaped crack in the fiber of a composite cylinder under remote uniform axial strain is presented. The solution is based on an axisymmetric geometry and satisfies all equations of elasticity. The stress intensity factor at the crack tip and the maximum stresses at the interface are examined to determine the nature of crack growth in the composite. These parameters are studied as a function of the elastic moduli of the fiber and the matrix, the interface properties, the fiber volume fraction and the crack length. The results obtained from this study are also compared with an equivalent model with planar assumptions. It is observed that the results obtained from the axisymmetric and planar models for a crack deflection criteria are similar.

**Title:** **Matrix Cracking in Brittle Matrix Composites with a Frictional Interface**

**Authors:** **A.K. Kaw, S. Kunchithpatham and N.J. Pagano**

**Journal:** **International Journal of Solids and Structures, Vol. 32, pp. 2127-2154, (1995).**

**Abstract:** The effect of a frictional interface on the response of a unidirectional ceramic matrix composite under a remote axial tensile strain and a temperature change is studied. The geometry of the composite is approximated by a concentric cylinder model with an annular crack in the axial plane of the matrix. The fiber-matrix interface follows the Coulomb friction law. On applying the boundary and the interface continuity conditions, the solution is obtained in terms of coupled integral and linear equations, and inequality conditions. The extent of the interfacial damage and the stress fields in the fiber and the matrix along the interface are studied for a SiC/CAS composite system as a function of coefficient of friction, temperature change and remote uniform axial strain. These results are also compared with a shear lag analysis model for an identical geometry and loading.

**Title:** Asymptotic Stresses Around a Crack Tip at the Interface Between Planar or Cylindrical Bodies

**Authors:** N.J. Pagano and A.K. Kaw

**Journal:** International Journal of Fracture, *in press*

**Abstract:** Closed-form equations are derived for the asymptotic stresses in the neighborhood of a crack tip impinging on an interface between two isotropic materials. The symmetric problem is considered and follows from an exact elasticity solution formulated by Gupta (1973). The equations are valid for the planar problem, where the interface is straight and also for an axisymmetric problem in the presence of an annular or penny-shaped crack. The equations may serve to establish a tentative criterion that defines the subsequent direction of a crack impinging on a bi-material interface. The ambiguity of the asymptotic stress state is highlighted and plausible application of the results is discussed.

**APPENDIX B. CUMULATIVE LIST OF RESEARCHERS**

The following students were involved in the research under this grant. Their thesis titles are given below and are available from

**University of South Florida Library,  
LIB 128, 4202 E. Fowler Avenue,  
Tampa, FL 33620.**

V. Bechel, "Effect of Nonhomogeneous Interphases and Global Fiber Volume Fraction on Mechanical Behavior of Composites", August 1993 (MS).

D. Jadhav, "Axisymmetric Elastic Response of a Composite Cylinder Containing a Fiber Crack", December 1993 (MS).

J. Ye, "Comparison of Axisymmetric and Planar Fracture Mechanics Models For Fiber Reinforced Composites", May 1994 (MS).

S. Kunchithapatham, "Matrix Cracking in Brittle Matrix Composites with a Frictional Fiber-Matrix Imperfect Interface", May 1994 (MS).